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WHEN IS A HEAVY QUARK NOT A PARTON?

Charged Higgs Production and Heavy Quark Mass Effects in the QCD-based Parton Model[†]

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Abstract

Several issues pertaining to the application of the QCD-based parton model to new physics processes involving heavy partons are described and quantitatively studied using charged Higgs boson production as a prime example. The naive parton model predictions are found to over-estimate the actual cross-section by a factor of 2 to 5, depending on the top-quark and Higgs masses. The role of the top quark as a "parton" is examined by a detailed study of the cancellation between the straight parton model contribution and a *subtraction* term required by QCD corrections. The accuracy of the commonly used zero-mass method for evaluating the *first-order QCD correction* is assessed (in light of the potentially large mass of the top quark) by a quantitative analysis of the cancellation of mass singularities between the correction terms. A pragmatic procedure for calculation based on a renormalization scheme without the heavy quark-parton is formulated and compared with the usual perturbative QCD formalism. The energy ranges over which heavy quarks (or other particles) should or should not be naturally treated as "partons" are delineated. Properly evolved parton distribution functions relevant to the specific renormalization schemes considered are employed for all the numerical studies in order to ensure consistency in the QCD framework.

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I. INTRODUCTION

The parton model has been remarkably successful in describing a wide variety of high energy processes involving some energy scales much larger than the masses of the known particles and the partons themselves. The theoretical basis of the parton model is provided by perturbative Quantum Chromodynamics (QCD).[1] In recent years, increasing attention has been directed toward applications of the "QCD-based parton model" to high energy processes involving heavy particles and heavy partons. Here the theory and the phenomenology are not as straightforward as for those involving only light (i.e. massless) particles such as embodied in the conventional parton model formalism. In studying physics phenomena beyond the currently available energy range these processes involving heavy particles are clearly of primary interest both within and beyond the Standard Model. A case in point is the production of Higgs particles, for which the heavy quarks play an important role.[2,3]

This work consists of a systematic study of the production of a heavy charged Higgs particle from initial state heavy quarks (in particular, the top-quark) as well as light partons (quarks and gluons). This process is of intrinsic interest;[4,5] and it provides a useful example to explore several important generic problems associated with new physics phenomena involving heavy particles. We shall study in detail the cancellations which take place between the *zeroth order* parton model contribution, the *first order* QCD correction to the hard cross-section, and the associated *subtraction term* which eliminates unwanted double-counting and removes potential mass-singularities. The near cancellation between the zeroth-order and the subtraction terms yields true cross-sections a factor of 2 to 5 (depending on the top and Higgs mass values) smaller than the straight parton-model results. This leads to the question: *when is a heavy quark not a parton*? Furthermore, a systematic study of the cancellation of mass-singularities between the first-order and the subtraction terms provides a quantitative assessment of the accuracy of the commonly used zero-mass method for calculating the QCD corrections to the cross-section. (This problem is of obvious concern in light of the potentially heavy mass of the top quark.) We shall examine a distinct renormalization scheme for calculating QCD-based parton model predictions which naturally leave out the heavy quark as a parton.[6] The numerical results on the actual range of validity of the various calculational schemes are important for many practical applications to new heavy particle production processes.[7]

The following sections (Part II) summarizes the general **theoretical background**. We discuss, in turn, (A) The "QCD-based Parton Model", (B) Leading Contributions to the Hard Cross-section, (C) Calculation of the QCD Correction Terms, (D) Elimination of Double Counting and Threshold Behavior, (E) Detailed Formulas for the Physical Cross-section, and (F) the Cancellation of Mass-singularities. Part III consists of concrete **numerical results** on the production cross-section, exploring its dependences on the unknown top quark mass, and on the renormalization scheme dependences which provide anwsers to the issues described in the previous paragraph. Specifically, the sub-sections are: (A) Threshold Behavior and the Cancellation of the Top-quark-parton Contribution, (B) Cancellation of Mass-singularities and Top-mass Dependence of the Hard Cross-section, and (C) A Pragmatic Renormalization Scheme with no Heavy Partons — Advantages and Limitations.

II. THEORETICAL BACKGROUND

We summarize those theoretical features of the QCD-based parton model that are relevant for its application to the production of heavy particles (which abound in all extensions of the Standard Model), especially those initiated by partons of non-negligible mass. The theoretical formulas introduced here will be used for numerical studies to be described in Part III. The results presented there will serve as concrete illustrations of the theoretical ideas discussed in this part and will specify the **range of validity** of previous qualitative expectations based on asymptotic considerations.

A. THE QCD-BASED PARTON MODEL

Consider the inclusive production of a charged Higgs particle H^+ in the collision of two high energy hadrons A, B:

$$A + B \to H^+ + X \tag{1}$$

where, as usual, X stands for "anything". We shall assume the Higgs coupling to quark-partons is given by the simple vertex expression

$$\Gamma = \frac{g_W}{2\sqrt{2}} \left[g_L(1-\gamma_5) + g_R(1+\gamma_5) \right] \tag{2}$$

where the overall coupling parameter g_W is the familiar gauge coupling constant of the SU(2) weak isospin. Since we shall focus on general features of heavy particle production in the parton model rather than particulars of Higgs models, the only detail of the couplings g_L and g_R we shall specify is that they are proportional to the mass of the quarks (scaled by M_W, say). Because of this last assumption, the heavy quarks play the dominant role in the Higgs production mechanism.

The QCD-based parton model is based on factorization theorems which, in the current case, can be stated succinctly as (cf. Fig. 1)

$$\sigma_{AB\to H} = \Sigma_{a,b}\ f_A^a \otimes \hat{\sigma}_{ab\to H} \otimes f_B^b \tag{3}$$

where a,b are parton labels which must be summed over, $\{f_A{}^a(x, Q)\}$ are <u>parton distribution functions</u> of parton species "a" in hadron "A", $\{\hat{\sigma}_{ab\to H}\}$ are <u>hard scattering cross-sections</u> involving only partons, and ⊗ represents a convolution integral — the precise form of which depends on the particular cross-section in question (e.g., σ_{total}, $d\sigma/dy$, $d\sigma/dp_T$, $d\sigma^2/dydp_T$, ... etc.). We shall present the precise expressions and detailed discussions in Section II.E. The pictorial representation of this factorized parton model formula is given in Fig. 1, where the " mark on the parton lines signifies that they are on-the-massshell and collinear to their respective parent-hadrons. The validity of the factorization theorem at high energies, Eq.(3), has been established to various degrees of rigor for different processes and for different kinematic regimes.[1] We shall assume it holds for the present case where the masses of the heavy quarks are taken to be much smaller than the overall center-of-mass energy $\sqrt{s}$.

The proof of the factorization theorem relies on perturbation theory. The factors on the right-hand side of Eq.(3) are defined in terms of Green's Functions which have the well-known attractive physical interpretations of parton distribution functions (f) and hard scattering cross-sections ($\hat{\sigma}$). **Precise definitions of these quantities are possible only within a given renormalization scheme R, and each quantity so defined also depends in general on an arbitrary renormalization scale parameter, usually denoted by μ.** The hard scattering cross-section $\hat{\sigma}$ can be reliably calculated in perturbation theory provided the scale parameter μ is chosen to be of the same order as a large energy scale relevant for the hard process, say M_H in the case under consideration. If only the lowest order term is kept in Eq.(3), then we obtain the naive parton model (with scale dependent parton distribution functions), and the question of choice of renormalization scheme (R) may be, and usually is, side-stepped. However, for the production of heavy particles via heavy partons the contributions from the naive "lowest order" diagram and "first order" corrections can be comparable for easily understandable reasons (cf. Ref. 6, and discussions on Fig. 2 in the next section). Thus, it becomes essential to specify the renormalization scheme that is used to define and calculate the relevant quantities which enter Eq.(3). It is well-known, of course, that even for processes involving only light particles any calculation of non-leading corrections, such as the numerically significant "K-factor" in Drell-Yan processes, requires a careful consideration of the choice of R. The case involving heavy particles poses, however, a different set of choices to be made and questions to be answered.

B. LEADING CONTRIBUTIONS TO THE HARD CROSS-SECTION

In our example, the "zeroth order" process contributing to the hard scattering comes from $t+\bar{b}\to H^+$ (cf. Fig.2a), assuming the usual quark species. The next order contributions are due to $g+\bar{b}\to\bar{t}+H^+$ (Fig.2b) and $t+g\to b+H^+$ (Fig.2c). If the t-quark represents the only "heavy parton", the relative order of magnitudes of the contributions from the three processes (Figs. 2a, 2b and 2c) to the overall cross-section (Eq.(3)) are expected to be $1 : 1 : \alpha_s$. Thus the contribution from Fig.2c can be neglected in practice. This result follows from the observation that, to a good approximation, the heavy quark distribution $f_A^t(x)$ can be obtained from the (first order) splitting of the gluon; it should be of order α_s with respect to the gluon distribution up until the full QCD evolution takes effect at an energy scale which is orders of magnitude larger than m_t. On the other hand, "light quark" distributions are generally considered to be of the same order of magnitude as the gluon distribution, since their masses are taken to be effectively zero. The case of the b-quark lies somewhere between these two extremes. For simplicity , we shall assume that the b-quark can also be treated as "light", hence its distribution function inside the hadron is of the same order of magnitude as that of the other light partons. A large part of this paper will be devoted to quantifying the reliability of this conventional wisdom.

The necessity to include the "first-order" process $g+\bar{b}\to\bar{t}+H^+$ along with the "zeroth-order" one, $t+\bar{b}\to H^+$, raises questions of **double counting**: the diagram Fig.2b contributes partially to the t-quark distribution function which is already included in the zeroth order calculation. This occurs when the intermediate t-quark line is on the mass-shell and collinear with the gluon line (hence, it corresponds to a **real parton** out of the gluon). This is also the configuration which gives rise to singularities of the scattering cross-section in the limit $m_t^2/s\to 0$ (hence the terminology <u>mass singularities</u>). An essential feature of the factorization theorem, Eq.(3), is that the hard cross-section $\{\hat{\sigma}_{ab\to H}\}$ is free from such mass singularities. Thus, the proper application of the "QCD-based parton model" requires a definite procedure to remove the mass-singularities; this, in turn, leads to the resolution of the double-counting problem.

C. CALCULATION OF THE QCD CORRECTION TERMS

Let $\sigma_{ab\to H}$ be the full cross-section for the **partonic process** a+b→H+X which is obtained by summing over Feynman diagrams such as those given in Fig. 2. We can determine the corresponding hard cross-section $\hat{\sigma}_{ab\to H}$ by the observation that **the full partonic cross-section $\sigma_{ab\to H}$ satisfies the same factorization theorem, Eq.(3), as the physical hadronic process (1).** In explicit terms, we have

$$\sigma_{ab\to H} = \Sigma_{c,d}\ f_a^c \otimes \hat{\sigma}_{cd\to H} \otimes f_b^d \tag{4}$$

where f_a^c is the distribution function of the c-parton inside the a-parton. This equation must hold order by order in perturbation theory — that is how the factorization theorem is proven in the first place. Since rules for calculating $\{ f_a^c \}$ and $\{ \sigma_{ab\to H} \}$ in perturbation theory are fairly well-known, Eq.(4) can be turned around to solve for the desired hard cross-sections $\hat{\sigma}_{ab\to H}$.

In lowest order perturbation theory (zeroth order in α_s),

$$^0f_a^c(x) = \delta_a^c\ \delta(x-1) \tag{5}$$

Eq.(4) then implies

$$\hat{\sigma}^0_{ab\to H} = \sigma^0_{ab\to H} \tag{6}$$

To first order in α_s, we have

$$\sigma^1_{ab\to H} = \hat{\sigma}^1_{ab\to H} + \hat{\sigma}^0_{ad\to H} \otimes {}^1f_b^d + {}^1f_a^c \otimes \hat{\sigma}^0_{cb\to H} \tag{7}$$

where the left-superscripts on the distribution functions, as well as the right-superscripts on the cross-sections, specify the order to which these quantities are to be calculated in perturbation theory. Inverting this equation and using Eq.(6), we obtain

$$\hat{\sigma}^1_{ab\to H} = \sigma^1_{ab\to H} - \sigma^0_{ad\to H} \otimes {}^1f_b^d - {}^1f_a^c \otimes \sigma^0_{cb\to H} \tag{8}$$

Now all the terms on the right-hand side are calculable, hence the hard cross-section $\hat{\sigma}^1_{ab\to H}$ is well defined. In particular,

$$\hat{\sigma}^1_{g\bar{b}\to H} = \sigma^1_{g\bar{b}\to H} - {}^1f_g^t \otimes \sigma^0_{t\bar{b}\to H} \tag{9}$$

Since there is no zeroth order process involving a gluon, the middle term in Eq.(8) is absent. Here ${}^1f_g^t$ is the first-order perturbatively calculated t-quark distribution inside the gluon.

Note that the second term on the right-hand side of Eq.(9) (to be called the subtraction term) has just the **collinear** configuration (signified by the convolution operation) discussed in Section II.B above. This can be represented graphically as shown in Fig.3 where the " sign on the t-quark propagator again signals that the momentum of that line is on-the-mass-shell and collinear to that of the parent gluon. **This term represents the overlap between the first-order and the zeroth-order contributions to the overall cross-section** (which requires its subtraction by the QCD formalism given above). This fact will become obvious in the next section (II.D). **The subtraction term also serves to remove the mass-singularity of the first order cross-section formula** in the limit $m_t \to 0$. This will be demonstrated in Section II.F.

D. ELIMINATION OF DOUBLE COUNTING AND THRESHOLD BEHAVIOR

Substituting Eqs.(6) and (9) in Eq.(3) we obtain for the overall cross-section

$$\sigma_{AB\to H} = f_A^t \otimes \sigma^0_{t\bar{b}\to H} \otimes f_B^{\bar{b}} - \tilde{f}_A^t \otimes \sigma^0_{t\bar{b}\to H} \otimes f_B^{\bar{b}} + f_A^g \otimes \sigma^1_{g\bar{b}\to H} \otimes f_B^{\bar{b}} + (A \leftrightarrow B) \tag{10}$$

where

$$\tilde{f}_A^t \equiv {}^1f_g^t \otimes f_A^g \tag{11}$$

The three terms on the right-hand side of Eq.(10) correspond to the "zeroth order", the "subtraction", and the "first order" terms respectively.

The particular way we wrote the subtraction term makes explicit its relationship to the zeroth order term on the right-hand side of Eq.(10). $\tilde{f}_g{}^t$ in Eq.(11) has a simple physical interpretation: it is the t-quark distribution in A resulting from the splitting of the gluons in A. The first two terms on the right-hand side of Eq.(10) can be grouped together to read:

$$\sigma_{AB\to H} = [f_A^t - \tilde{f}_A^t] \otimes \sigma^0_{t\bar{b}\to H} \otimes f_B^{\bar{b}} + f_A^g \otimes \sigma^1_{g\bar{b}\to H} \otimes f_B^{\bar{b}} + (A \leftrightarrow B) \tag{12}$$

We see that, by requiring the presence of the subtraction term, **the QCD formalism explicitly removes from the leading term the first-order QCD contribution to the on-shell collinear parton distribution function when the full first-order correction term is added.** This avoids the potential **double counting** in a simple-minded application of the naive parton model. One can see this point graphically by attaching Fig.3 to Fig.1 and comparing with the corresponding contribution from Fig.2a.

We anticipate that the subtration term nearly cancels the zeroth order term for energy scales just above the top-quark mass. This is because the t-quark distribution function $f_A{}^t$ is well approximated by the perturbative distribution $\tilde{f}_A{}^t$ at energy scales of this order. The cancellation of the top quark-parton contribution (Fig.2a) by the substraction term effectively removes the t-quark as a "parton". In this region the true production cross-section will be close to that given by the first order term alone. However, it is not obvious what is the extent of the energy range over which this is true. We shall examine this problem numerically in Section III.A. In view of this cancellation, it is natural to ask whether one can legitimately avoid calculating the first two terms over this region for all practical purposes. This question will be addressed in Section III.C where a pragmatic renormalization scheme is introduced, and both its usefulness and limitations will be discussed.

E. DETAILED FORMULAS FOR THE PHYSICAL CROSS-SECTION

We now give the explicit expressions for the unabbreviated formulas. We shall start with the formula for the total cross-section, which has the basic form

$$\sigma^{tot}_{AB}(s,\ldots) = \int d\xi_a d\xi_b \; f^a_A(\xi_a)\; \hat{\sigma}^{tot}_{ab}(\hat{s}=\xi_a\xi_b s,\;\ldots)\; f^b_B(\xi_b) + (A \leftrightarrow B) \tag{13}$$

[cf. Eq.(3)], where for simplicity we have dropped the symbols ($\rightarrow$H) after the initial particle labels on σ and $\hat{\sigma}$. Applying this to the right-hand side of Eq.(10), and working out the appropriate one- and two-particle kinematics, we can uncover the contributions of the three terms to the differential cross-section $d\sigma/dy$, where y is the rapidity of the Higgs particle:

(i) for the <u>zeroth order term</u> we obtain:

$$\begin{aligned}\frac{d}{dy}\sigma^0_{AB} &\equiv f^t_A \otimes \frac{d}{dy}\sigma^0_{t\bar{b}} \otimes f^{\bar{b}}_B \\ &= \frac{\pi}{24s}\, g^2_W(g^2_L+g^2_R)\;[f^t_A(x_a,M_H)\; f^{\bar{b}}_B(x_b,M_H) + (A\leftrightarrow B)]\end{aligned} \tag{14}$$

where

$$x_a = \frac{M_H}{\sqrt{s}}\, e^{y}, \qquad x_b = \frac{M_H}{\sqrt{s}}\, e^{-y} \tag{15}$$

(ii) for the <u>first order hard scattering term</u> the result is:

$$\begin{aligned}\frac{d}{dy}\sigma^1_{AB} &\equiv f^g_A \otimes \frac{d}{dy}\sigma_{g\bar{b}} \otimes f^{\bar{b}}_B = \frac{g^2_W\alpha_s}{96s}\,(g^2_L + g^2_R) \\ &\int\frac{d\xi_a}{\xi_a}\int\frac{d\xi_b}{\xi_b}\int dp^2_T\,\delta[s(\xi_a-x_a)(\xi_b-x_b)-m^2_t-p^2_T]\cdot[f^g_A(\xi_a)\,|T_{g\bar{b}}|^2 f^{\bar{b}}_B(\xi_b) + (A\leftrightarrow B)]\end{aligned} \tag{16}$$

where the squared reduced matrix element is given by

$$\begin{aligned}|T_{g\bar{b}}|^2 &= \frac{\hat{s}}{m^2_t-\hat{u}}\left[1 - 2\frac{M^2_H-\hat{u}}{\hat{s}}\left(1 - \frac{M^2_H-m^2_t}{\hat{s}}\right)\right] \\ &+ 2\,\frac{m^2_t}{(m^2_t-\hat{u})}\left(1 - \frac{M^2_H-\hat{u}}{m^2_t-\hat{u}}\right) + \frac{m^2_t-\hat{u}}{\hat{s}}\end{aligned} \tag{17}$$

and $(\hat{s},\hat{t},\hat{u})$ are the Mandelstam variables for the partonic process $g+\bar{b} \rightarrow H^{+}+\bar{t}$.

(iii) In order to compute the subtraction term in Eq.(10) [cf. also Eq.(9)], we need the perturbative distribution function ${}^{1}f_{g}^{t}(x,\mu)$. This is obtained by evaluating the Feynman diagrams contributing to the t-quark distributions inside the gluon. (Cf. Ref. 8 for precise definitions and Feynman rules.) As divergent integrals are encountered in these perturbative calculations, one must apply a subtraction to obtain the renormalized parton distribution function. Using MS-bar subtraction, we find

$$ {}^{1}f_{g}^{t}(x,\ \mu) = \frac{\alpha_s}{4\pi}[(\log \frac{m_t^2}{\mu^2} +1)\ (1-2x + 2x^2) + O\ (\frac{m_t^2}{\mu^2})] \tag{18} $$

Thus the term singular in the $m_t \rightarrow 0$ limit is,

$$ \tilde{f}_{A}^{t}(x,\ \mu) = \frac{\alpha_s}{4\pi} \cdot \log \frac{m_t^2}{\mu^2} \cdot \int_x^1 \frac{d\xi}{\xi}\ (1-2\xi + 2\xi^2)\ f_A^g(\frac{x}{\xi},\ \mu) \tag{19} $$

and the *subtraction term* in Eq. (10) is

$$ \begin{aligned} \frac{d}{dy}\ \sigma_{AB}^{s} &= \tilde{f}_A^t \otimes \frac{d}{dy}\ \sigma_{t\bar{b}}^{0} \otimes f_B^{\bar{b}} \\ &= \frac{\pi}{24s}\ g_W^2(g_L^2 + g_R^2)[\tilde{f}_A^t(x_a,\ M_H)\ f_B^{\bar{b}}(x_b,\ M_H) + (A\leftrightarrow B)] \end{aligned} \tag{20} $$

It is easily verified that the explicit formula for $\tilde{f}_A{}^t$, Eq.(19), is an approximate solution to the QCD evolution equation (to order α_s) for $f_A{}^t$, provided $\log(m_t/\mu)$ is not too large, as anticipated in Sec.II.D.

This completes the detailed formulas for calculating the inclusive y-distribution of the Higgs particle. The physical cross-section, according to Eq.(10), is the sum of Eqs.(14) and (16) with Eq.(20) subtracted. We note that **these individual terms are renormalization scheme dependent**. The net result, however, should be independent of the choice of the renormalization scheme. The relative sizes of the various terms can vary considerably from scheme to scheme. We shall evaluate these terms numerically in Part III and discuss the relative merit of renormalization schemes in Section III.C.

F. THE CANCELLATION OF MASS-SINGULARITIES

It is not hard to see that the first order cross-section σ^1 and the subtraction term σ^S both have mass singularities. We would like to check explicitly that their singular terms cancel in Eqs.(9) and (10), so that the hard cross-section $\hat{\sigma}_{g\bar{b}}$ as well as the physical cross-section σ_{AB} are well-behaved in the limit $m_t \rightarrow 0$.

The (logarithmic) mass singularity in the subtraction term σ^S, as given by Eqs.(19) and (20) is manifest. This is not so in the first order cross-section term σ^1; the singularity there arises implicitly from the fact that the squared matrix element, Eq.(17), contains the factor $(m_t-\hat{u})^{-1}$ which diverges as $p_T\to 0$ in the limit $m_t \to 0$. It is not hard to extract this singular term from Eqs.(16) and (17) — one can use the δ-function in Eq.(14) to do the ξ_b-integration, then Taylor-expand all functions in the integrand which are regular in p_T [in the $m_t \to 0$ limit] with the exception of the singular factor $(m_t^2 + p_T^2)^{-1}$ [arising from $(m_t^2-\hat{u})^{-1}$] which is treated exactly. One can verify explicitly

$$\lim_{m_t\to 0} \left[\frac{d}{dy}\sigma^1_{AB} - \frac{d}{dy}\sigma^s_{AB} \right] = \text{finite} \tag{21}$$

(The detailed expression on the right-hand side is not significant.) **Thus the subtraction term removes the mass singularity in the first order cross-section (in the limit $m_t\to 0$) as well as avoids the double counting** of the naive parton model picture.

Although $(d\sigma^1/dy - d\sigma^S/dy)$ is regular in the limit $m_t \to 0$, **it is not independent of m_t**. In particular, if m_t is not too small compared to the large energy scales of the problem, such as M_H, this quantity may differ substantially from its value in the $m_t\to 0$ limit. When the condition $m_t/M_H \ll 1$ is satisfied, this correction term to the lowest order formula will be well-approximated by its zero-mass limit. In that case, it is the common practice to calculate $d\sigma^1{}_{AB}/dy$ directly in the m_t=0 theory using minimum subtraction to regulate the divergence **and** to remove the mass-singularity.[5] (This obviates the need to calculate the substraction term separately.) The equivalence of this method to the $m_t\to 0$ limit of the general procedure described in this paper is usually not explicitly demonstrated. In any case, **if m_t/M_H is not very small the zero-mass method is not relevant a priori.** This is important if the top quark is, indeed, heavy as all current experimental results seem to indicate. We shall study this point numerically in Section III.B to determine the range of validity of the zero-mass method.

III. NUMERICAL RESULTS

Numerical calculations presented below are based on the formalism of the previous section using parton distribution functions generated by a QCD evolution program with a variety of input top quark mass and other parameters as necessary for the analyses. It is essential to generate the appropriate parton distribution sets for the various aspects of this investigation according to the rules of QCD rather than to use the commonly available fixed-parameter sets[4,9] because the latter were not designed to incorporate variable heavy quark masses and renormalization-scheme dependence both of which are of primary importance to our study. For definiteness, all calculations presented below are done for proton-proton scattering at a total CM energy of 40 TeV.

A. THRESHOLD BEHAVIOR AND THE CANCELLATION OF THE TOP-QUARK-PARTON CONTRIBUTION

In Figs.(4a-c)) we show the differential cross-section of charged Higgs production at rapidity y = 0 of the produced Higgs particle plotted as functions of Higgs mass for three values of assumed top quark masses — 40, 80, and 300 GeV respectively. The calculation is performed with parton distributions generated in the conventional scheme[10] with all six quark flavors active above their mass thresholds. The same top quark mass enters the

QCD-evolved parton distribution functions and the hard cross-section calculations. In the hard cross-section calculation, the top quark mass is kept explicitly in both the first-order and the subtraction terms. (Hence, we are **not** using the zero-mass approximation as most similar works[5] do.) Initial parton distributions at $Q_0 = 2.25$ GeV are taken from EHLQ.[4]

Four curves are displayed in each graph corresponding to contributions from the "zeroth-order" (labelled SIG-0), the "first-order" (SIG-1), the subtraction term (SUBTR), and the "true" cross-section (SIG-T), i.e. the sum of the three. The general feature shared by the three graphs is that, as anticipated in Section II.D, **the zeroth-order contribution is rather closely tied to (hence cancelled by) the subtraction term from threshold all the way to 1 TeV and beyond in the Higgs mass.** As a consequence, the "first-order" contribution alone provides a good approximation to the true cross-section over much of this range. Note that the zeroth order contribution alone can be about twice the true cross-section at $m_t = 40$ GeV, and this factor increases to five for $m_t = 300$ GeV. It is obvious that for increasingly heavy top quark, the naive parton picture (i.e. the zeroth order term) becomes a progressively worse representation of the true physics.

According to Eq.(12), the observed cancellation results from the fact that, over this energy range, the full t-quark distribution function $f_A{}^t$ (obtained from solving the QCD evolution equation) is well approximated by $\tilde{f}_A{}^t$ (the convolution of the first-order perturbative distribution of the t-quark inside the gluon with the gluon distribution function inside the hadron; cf. Eq.(11)) We show this cancellation in more detail near the threshold for a typical case ($m_t = 80$ GeV) in Fig.5. For this calculation, we set the masses of all the light quarks (including the b-quark) to zero in both the properly evolved parton distribution function and the hard cross-section formula. This allows the physical threshold for Higgs production to go down all the way to the top quark mass near which the full and the perturbative distribution functions rigorously coincide. The numerical results confirm the cancellation mechanism discussed in Sections II.D and II.F in every detail.

Although the threshold behavior has been expected on theoretical grounds (a detailed discussion has been given, for instance, in Ref.6), the wide range over which the near cancellation takes place has not been clearly established previously. Since the contribution due to the top quark-parton (SIG-0) is almost cancelled by the subtraction term (SUBTR), **the QCD theory is telling us that over this entire range the top quark is really not a "parton"**. Effectively, only the subprocess involving the light partons (SIG-1) makes a meaningful contribution to the true cross-section. In light of this result, it is natural to seek a simple theoretical basis for the effective dominance of the first-order contribution to the production cross-section without the redundant cancelling terms. This is done with the choice of an alternative renormalization scheme as discussed in Section III.C.

At first glance, it may appear somewhat counter-intuitive that the "first-order correction term" in a parton process can yield a much better approximation to the true cross-section than the "zeroth-order contribution" (which corresponds to the naive parton model expectation). If this holds true over most of the energy scales reported above, it is natural to ask: at what energy range and for what heavy parton masses will the "zeroth-order term" become truly dominant, and the naive parton picture become a good description of the physics? Or, in other words: **when will the top quark behave like a parton?** The trends seen in Figs.(4a-c) suggest that we should look in the region of large s, M_H and small m_t, as is also obvious from theoretical considerations. We examine a case for the same $\sqrt{s}$ (i.e. 40 TeV) but artificially small m_t. Fig.6 shows the same curves as above, but for the case of an assumed top quark mass of 5.0 GeV (just above the bottom mass). Indeed, by going to this extreme we see the onset of the naive parton model result. The contribution due to the subtraction term steadily moves away from the zeroth order term toward the first-order term, and the true cross-section interpolates between the first-order contribution (near the threshold) and the zeroth-order contribution (at the high Higgs mass end). These results show that even if 5 - 10 GeV quark masses appear small on the scale of $\sqrt{s}$ and $\sqrt{\hat{s}}$, one cannot freely apply the naive parton model formulas for processes initiated by these quarks even at the highest energy range planned for SSC. **In fact, Fig.6 shows that in the SSC energy regime, a 5 GeV quark is neither "light" nor "heavy": the true cross-section interpolates between the straight parton result (SIG-0) for very large M_H and that of a heavy-parton (SIG-1) for M_H of the same order as m_t.**

In Fig.7 we present a typical plot of the rapidity distribution of the produced Higgs particle, showing the true differential cross-section as well as individual contributions from the three perturbative QCD terms described above. The shape of the distribution is very similar for all terms; the relative magnitudes at arbitrary y are therefore well represented by that at y = 0 as shown in Figs.(4a-c) for a wide range of M_H and m_t.

B. CANCELLATION OF MASS-SINGULARITIES AND TOP-MASS DEPENDENCE OF THE HARD CROSS-SECTION

Next, we investigate quantitatively the cancellation of mass-singularities between the first-order and the subtraction terms and to answer the question: **how good is the zero-mass approximation for the QCD correction term used in most calculations of this type**? (This is usually done by applying dimensional regularization to the zero-mass theory.) We could directly calculate the two terms in Eq.(9) for the hard cross-section as functions of m_t and examine the $m_t \to 0$ limit. Instead, we include the hard cross-section in the convolution integral for the physical scattering cross-section using fixed parton distribution functions in order to compare these results with the zeroth order contribution to the cross-section. The latter involves a different parton process, hence cannot be compared with the first-order terms quantitatively at the parton cross-section level.

A typical case is shown in Fig.8. The mass of the Higgs particle is 300 GeV and the fixed top-quark distribution correspond to EHLQ-set 1.[4] The differential cross-section at y = 0 is plotted against the variable top quark mass used in the calculation of the hard cross-section, Eq.(9). The horizontal line labelled SIG-0 is the mass-independent zeroth-order contribution. The first-order and the subtraction terms, represented by the dashed (SIG-1) and dotted (SUBTR) curves respectively, are seen to rise sharply with decreasing m_t as expected from their mass-singularities. Their difference, however, approximately approaches a constant below m_t = 10 - 20 GeV. The cancellation of the mass-singularity is clearly seen. The solid line labelled as SIG-T is the overall cross-section given by Eq.(10). The horizontal line labelled SIG-M0 is the overall cross-section one obtains by using the $m_t = 0$ limit of the first-order QCD corrections (i.e. the difference between SIG-1 and SUBTR).

The limiting value of the difference described above corresponds to the *first-order QCD correction* to the production cross-section obtainable in a direct *zero-mass calculation* (i.e. $m_t = 0$) of that quantity using dimensional regularization techniques.[5] **Since $m_t \neq 0$, one must ask: how reliable is this zero-mass approximation to the cross-section?** The answer clearly depends on the physical top quark mass which is not known at present (except that it is likely to be quite a bit heavier than previously thought). From the graph, we can see that the true cross-section is about 30% larger than that obtained with the zero-mass method if m_t = 80 GeV, and 200% larger if m_t = 200 GeV. Similar results are obtained for other values of the Higgs mass.

C. A PRAGMATIC RENORMALIZATION SCHEME WITH NO HEAVY PARTONS — ADVANTAGES AND LIMITATIONS

The results of Sec.III.A, particularly the near equality of the *first-order* contribution (with physical top quark mass) and the full cross-section over most of the parameter ranges, suggest that we seek a theoretical framework in which this result occurs naturally — without the complication of the other two cancelling terms. In such a framework, calculations of the physical cross-section will be considerably simplified.

This is indeed possible if one takes advantage of the renormalization scheme dependence of perturbative QCD, and choose a scheme in which the equivalent of the above **"first-order" term represents the only contribution to the cross-section.** Crudely speaking, in such a scheme the "heavy quark" will be treated rather differently from the other quark-partons. In fact, it will not be considered as a parton at all; any effect due to this heavy quark will be calculated explicitly, in contrast to that of the other partons (for which all soft physics is summed together and factorized into parton distribution functions).

Technically, such a scheme can be realized by the following prescription for removing divergences in general Feynman diagrams: all diagrams with no "heavy quark" lines are regularized by minimal (or MS-bar) subtraction; and all other diagrams by, say, BPHZ subtraction.[11] In order for the simplification mentioned above to occur, **this prescription must be applied to the full range of renormalization scale above the "light quark" mass thresholds — including that above the "heavy quark" mass threshold.** This is in contrast to the more conventional schemes[10] (such as the one used in the calculations of Sec.III.A) where every quark is turned into an active "parton" somewhere above its mass threshold.

What are the limitations of this simplified scheme? Since Feynman diagrams involving the heavy quark are not summed (hence no corresponding distribution function is factored out), they must be taken into account individually. Consequently, correction terms to the leading contribution (Fig.2b in our example) in this scheme will be of order $\alpha_s \cdot \log(m_t/\mu)$ where μ is the renormalization scale — rather than α_s as for corrections to light parton contributions. (The logarithm factor, of course, arises from loop subdiagrams.) In our problem $\mu \approx M_H$. Thus the range of validity of this scheme is limited to the region where $\alpha_s \cdot \log(m_t/\mu) \ll 1$. This is the same range over which the heavy quark distribution in the complete scheme is well approximated by its low-order perturbative expression (e.g. $f_A^{\ t}$ and $\tilde{f}_A^{\ t}$ as discussed in Section II.D). The numerical results presented in Section

III.A indicate that this covers most the energy range of current interest provided the degree of accuracy required is not severe.

We have performed a numerical calculation of the charged Higgs cross-section in this simplified scheme and have compared the results to the complete calculation of Sec.III.A. The appropriate parton distribution functions for this calculation are generated separately according to the new renormalization scheme (5 effective parton flavors for all energies above the bottom threshold). In order to make the comparison meaningful we use a Λ_{QCD} value which gives the same effective running coupling α_s as in the conventional scheme over the energy range of 5 - 20 GeV — where α_s has been measured. The gluon and bottom distribution functions obtained this way differ slightly from those of the conventional scheme (by varying amounts, depending on x and Q) due to the absence of the top quark-parton in the new scheme. The difference is not significant over the energy range of interest, as one may expect (the top distribution is numerically small). Fig.9 shows the comparison for the case m_t = 80 GeV over the range 85 GeV < M_H < 180 GeV. The curve labelled NTP-RS is the cross-section from the No-Top-Quark (as "parton") Renormalization Scheme; the ones labelled SIG-1 and SIG-T are the same ones as in Fig.4b from the general renormalization scheme. (Note the linear scale here vs. the log scale in Fig.4.) The difference between SIG-1 and NTP-RS lies between 15 to 25%. The "true cross-section" is closer to the NTP-RS curve near threshold, but moves toward SIG-1 at higher M_H.

We conclude from this exercise that, in the (limited) energy range of up to 1 - 10 TeV, the simplified renormalization scheme provides a convenient way to estimate cross-sections; Fig.9 indicates that the accuracy of using this calculational scheme is in the range 20 - 30 %.

IV. CONCLUSION

We have presented a systematic study of the production of charged Higgs particle in the framework of the QCD-based parton model. A number of general issues pertaining to new processes initiated by heavy quark-partons have been examined in detail. For energy scales up to a few TeV, a top quark of mass above, say, 20 GeV largely behaves like a *heavy particle*, hence is more naturally treated distinctly from the *partons* of the conventional perturbative QCD formalism. The quantitative analysis of the cancellation of the heavy parton contribution by the subtraction term and the discussion of the alternative renormalization scheme without the heavy parton provide the theoretical bases for this increasingly recognized pragmatic approach to heavy particle production. We have also examined the cancellation of mass singularities between the first order QCD correction terms to the parton model formula. This calculation allowed an assessment of the reliability of the often used zero-mass method for evaluating the QCD correction to this type of processes.

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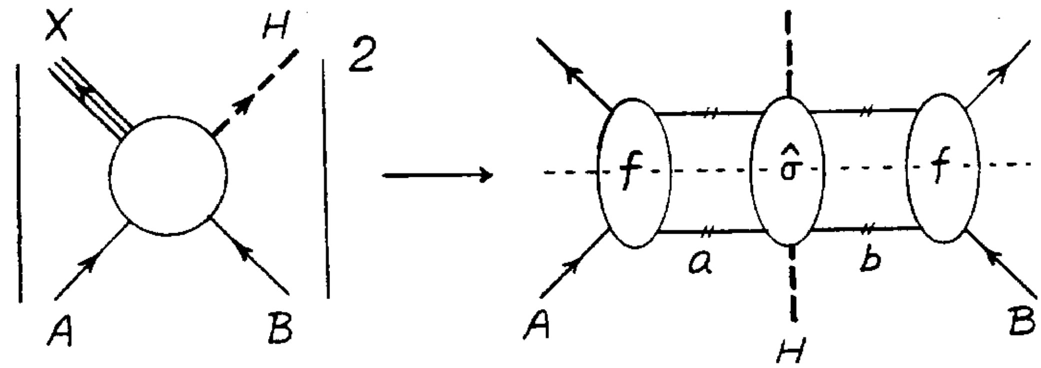


Fig.1 Graphical representation of the Factorization Theorem of QCD which provides the theoretical basis of the Parton Model.

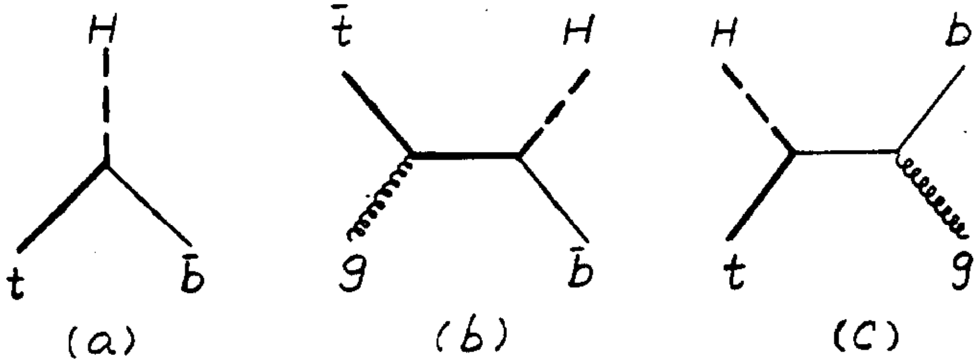


Fig.2 Partonic processes contributing to the production of charged Higgs particle production: (a) *zeroth order*, (b) and (c) *first order* in the QCD coupling.

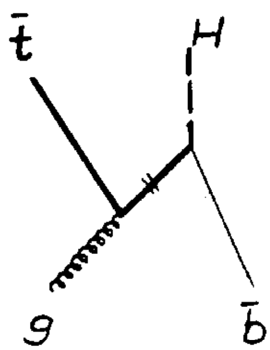


Fig.3 Graphical representation of the first-order QCD *subtraction term* to the hard scattering. The intermediate t-quark line is on-the-mass-shell and collinear to the gluon line.

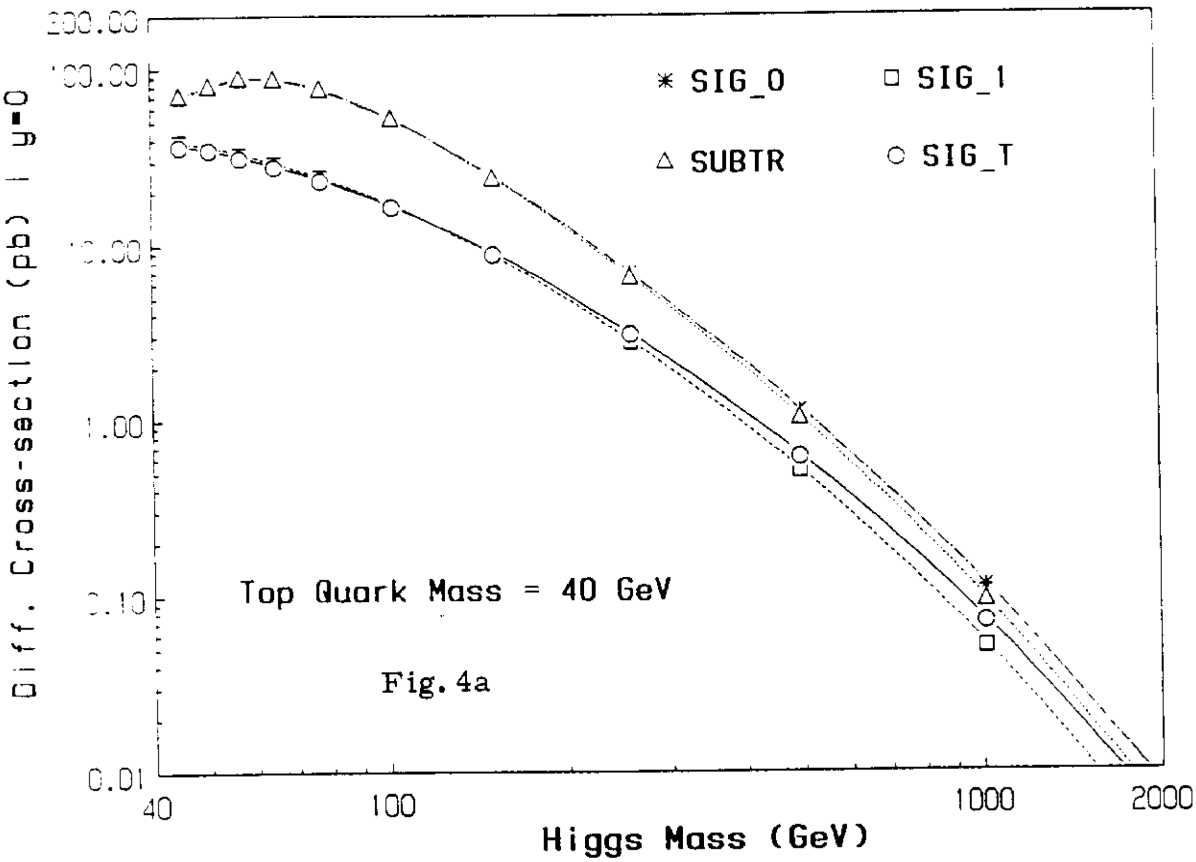
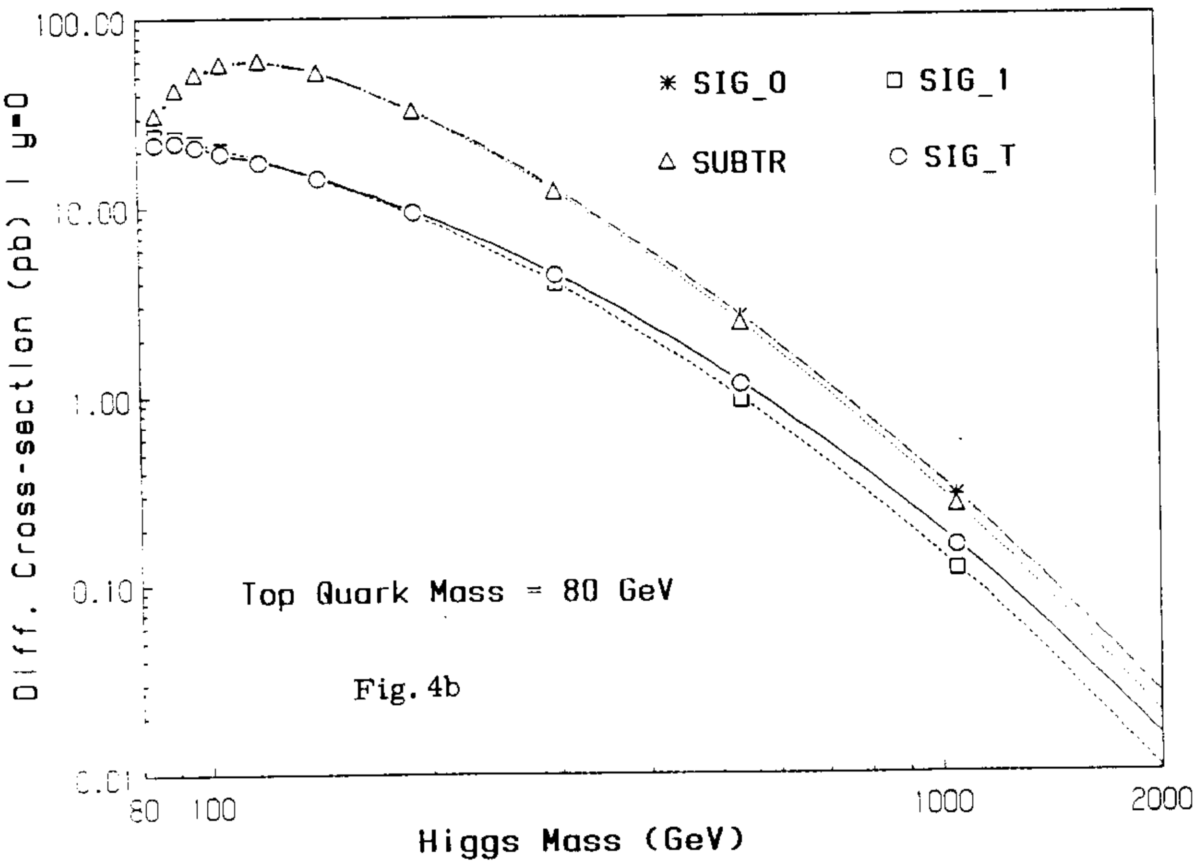

200.00
100.00
10.00
1.00
0.10
0.01
40
100
1000
2000
Diff. Cross-section (pb) | y=0
Higgs Mass (GeV)
* SIG_0
□ SIG_1
△ SUBTR
○ SIG_T
Top Quark Mass = 40 GeV

Fig. 4a


100.00
10.00
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80
100
1000
2000
Diff. Cross-section (pb) | y=0
Higgs Mass (GeV)
* SIG_0
□ SIG_1
△ SUBTR
○ SIG_T
Top Quark Mass = 80 GeV

Fig. 4b

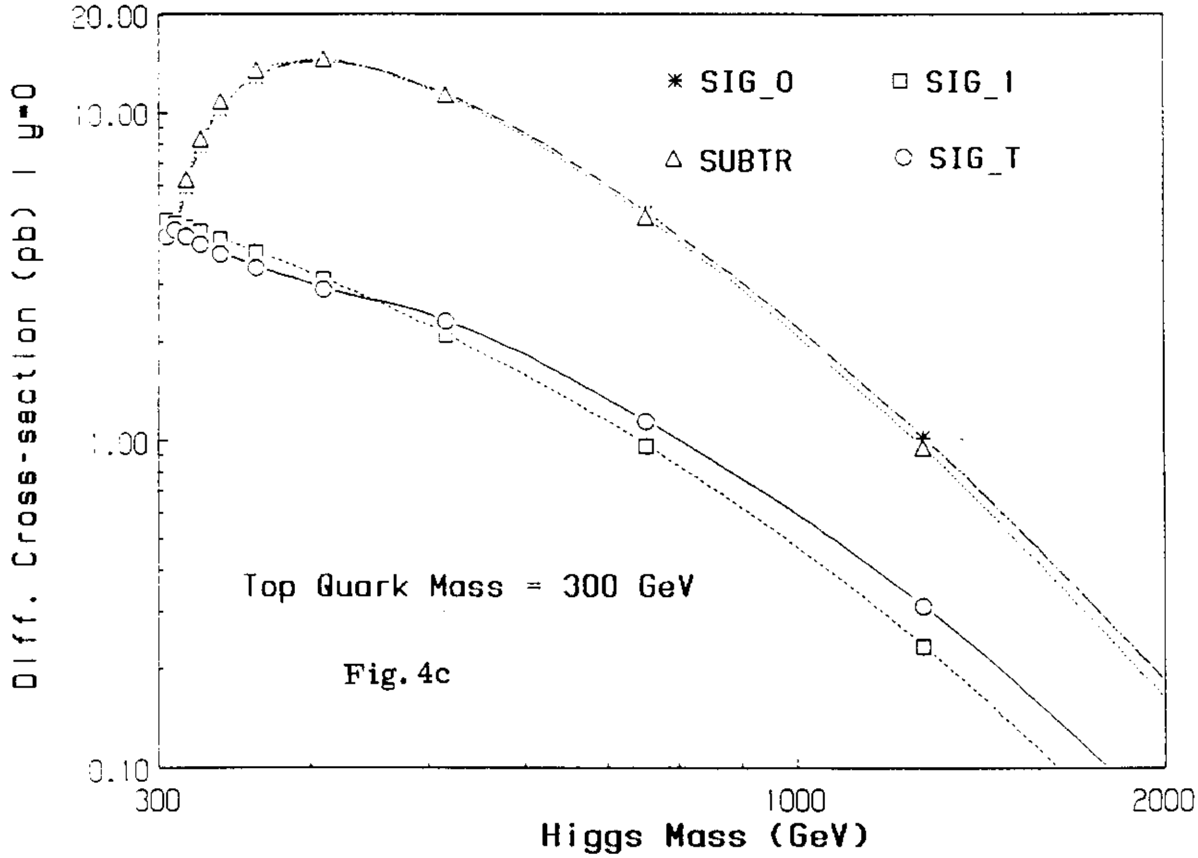

Fig.4 Charged Higgs production cross-sections $d\sigma/dy$ at y=0 as functions of the Higgs mass for three different values of the top-quark mass as indicated. The four curves on each graph correspond the *straight parton-model* (SIG-0), the *first-order* (SIG-1), the *subtraction term* (SUBTR), and the *true* cross-sections (SIG-T) respectively.

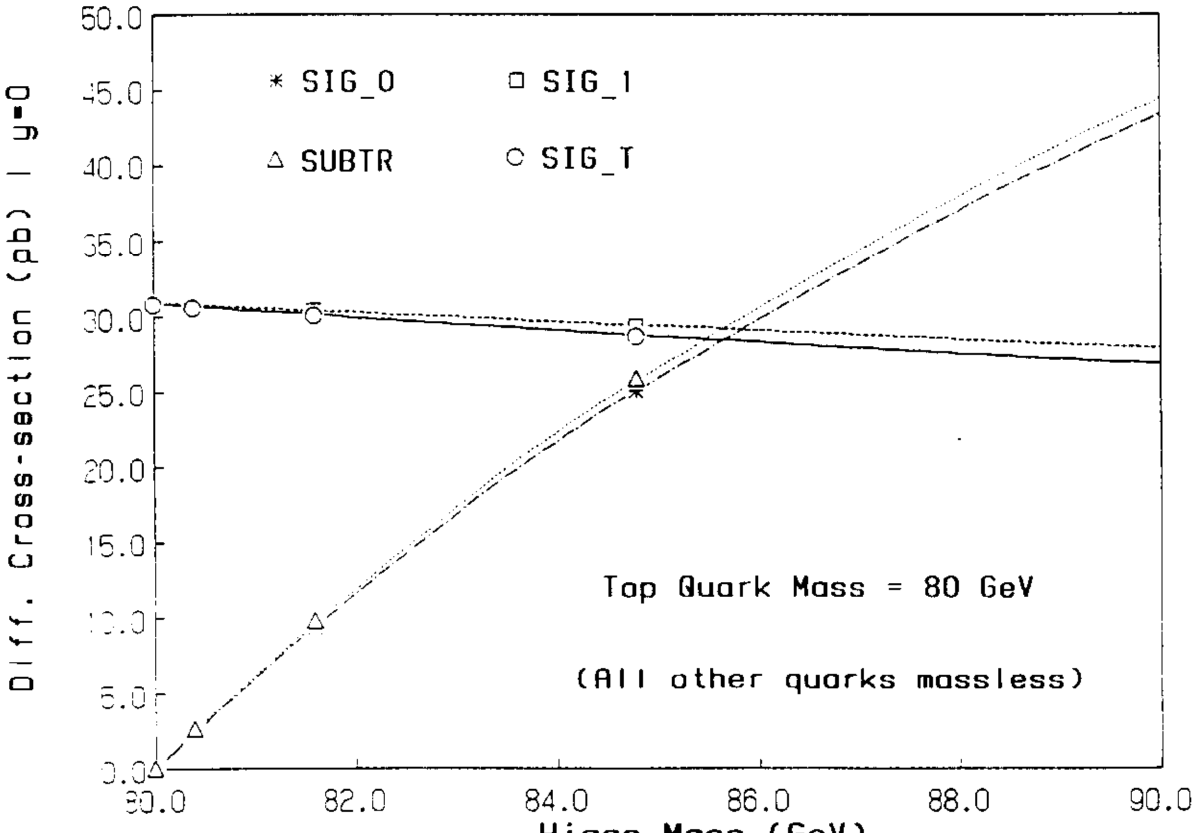

Fig.5 The cancellation between the straight parton-model contribution and the subtraction term just above the mass threshold of the top quark.

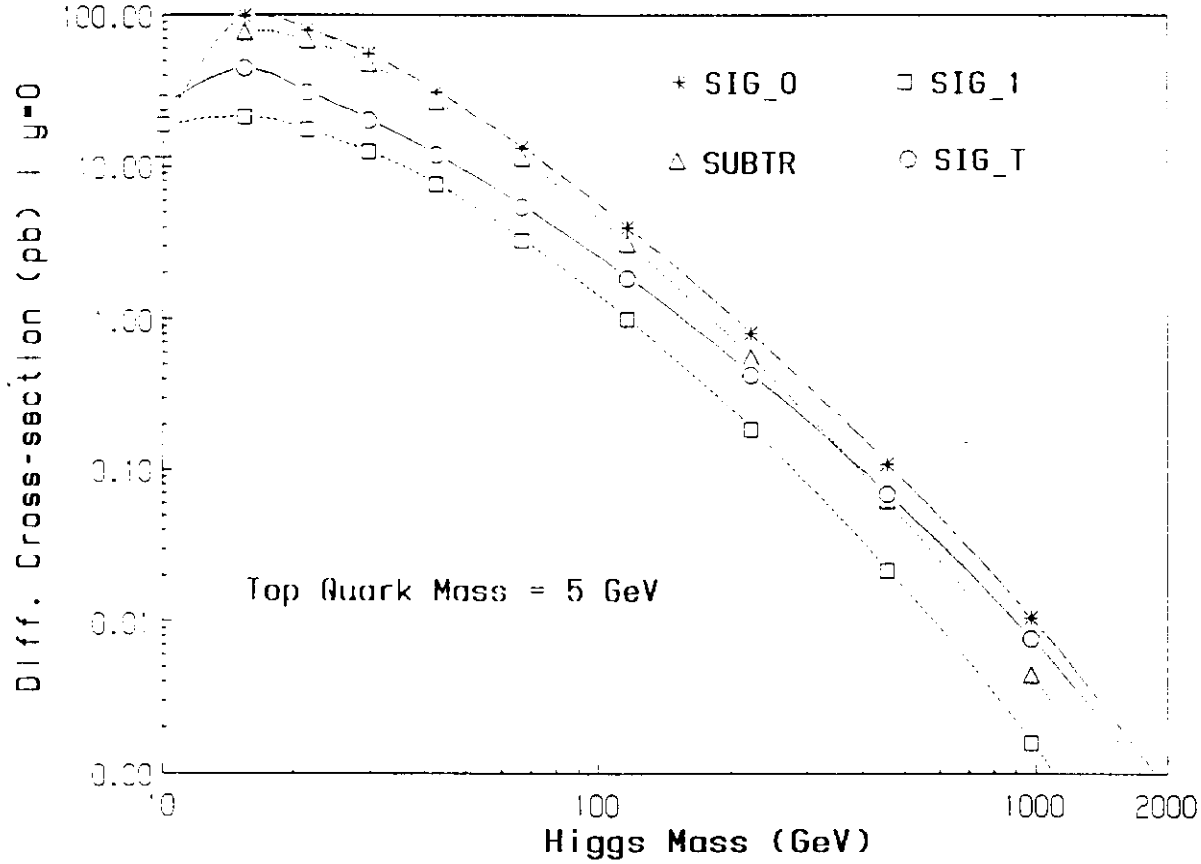

Fig.6 Charged Higgs production cross-section for an assumed 5 GeV top quark mass. Labels are the same as in Fig.4. The true cross-section lies in between SIG-0 and SIG-1.

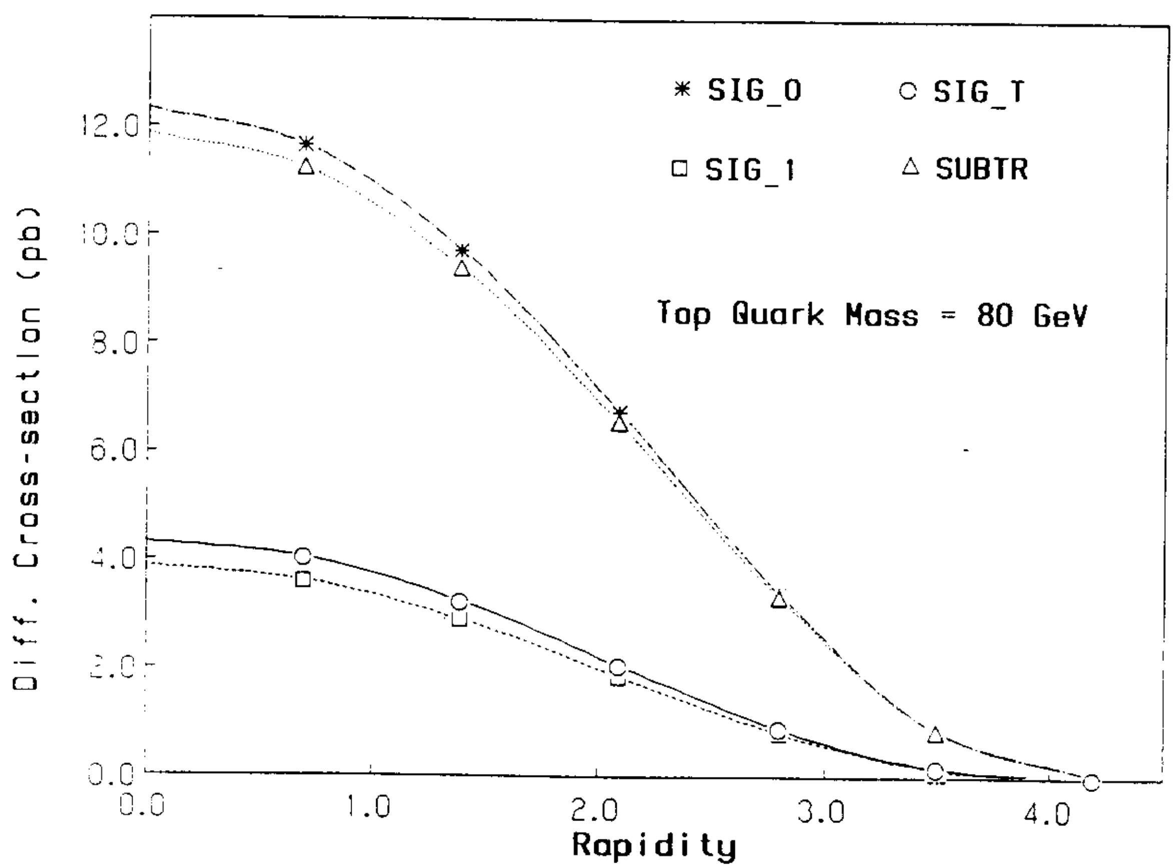

Fig.7 Rapidity distribution of the produced Higgs particle for the individual terms in the QCD-based parton formula and for the true cross-section.

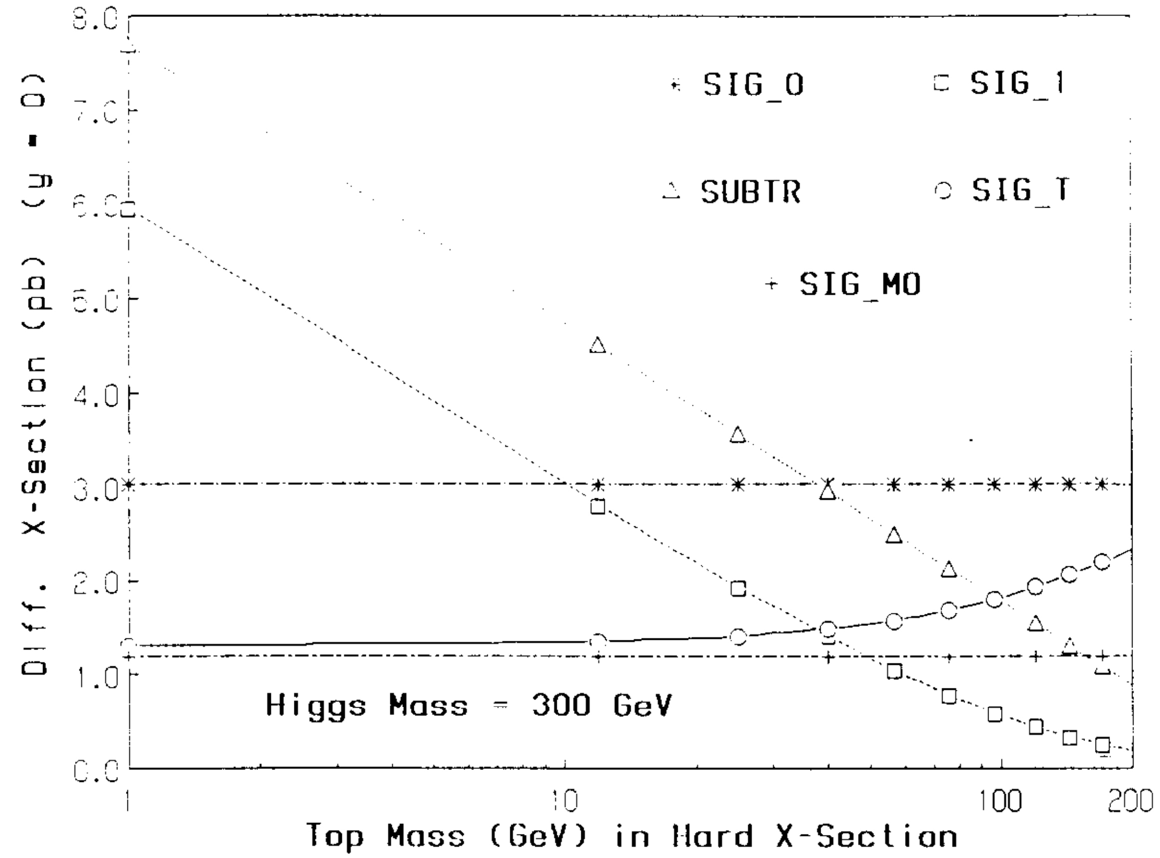

Fig.8 The cancellation of mass-singularities between the first-order QCD correction terms; and comparison of the *zero-mass approximation* (SIG-M0) of the QCD correction to the exact result.

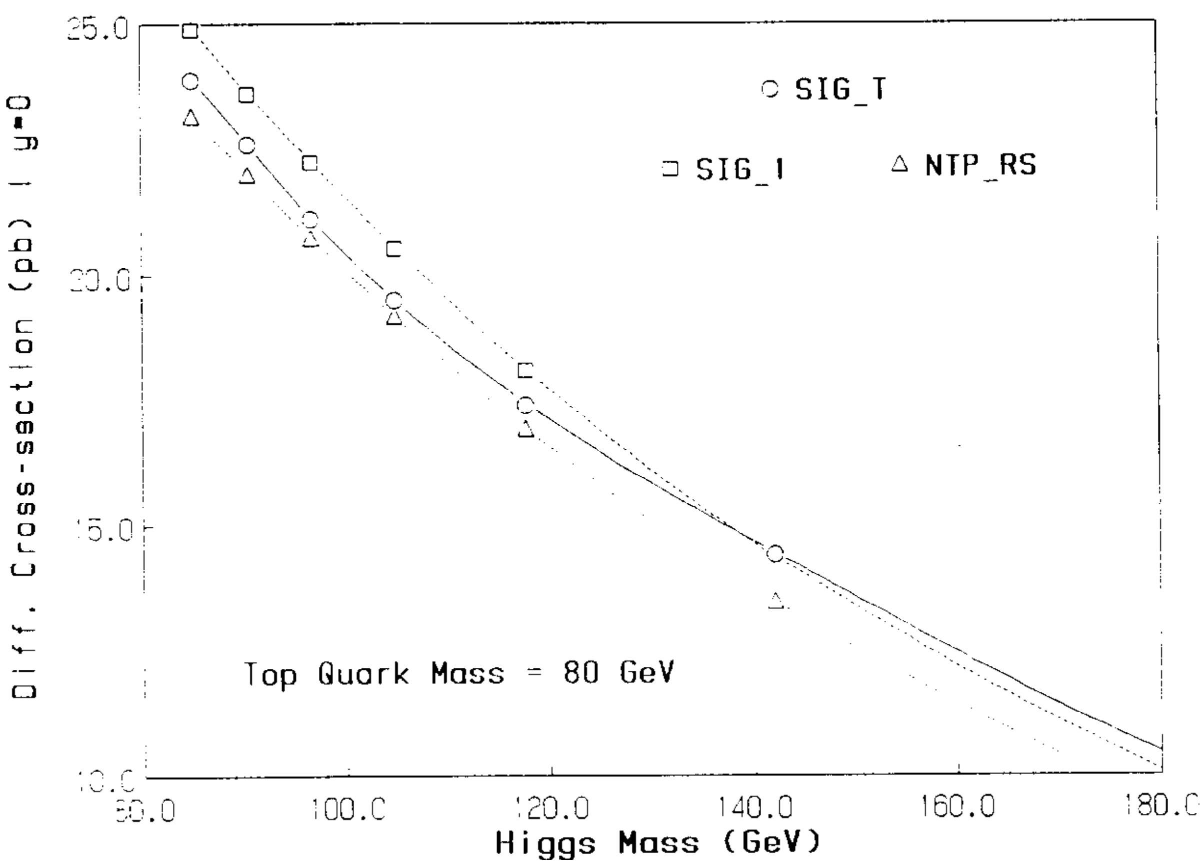

Fig.9 Comparison of Higgs production cross-sections calculated in the two renormalization schemes described in the text.